Math 72 9.5 - 3rd

Math 62 11.5 - 3rd

Mixed practice
Exponential Equations
& Log Equations

Math 70 Mixed Practice 9.8

Solve for exact answers, then find approximations with four decimal places.

1) $\ln(3x) - \ln(x-3) = 2$

2) $3^{2x} = 7$

3) $\log_5 2 + \log_5 x = 2$

4) $3^{2x+1} = 6$

5) $\log(5x) - \log(x+1) = 4$

6) $5^{3x-5} = 4$

7) $\log_2 x + \log_2 2x - 3 = 1$

8) $3 \cdot 4^{x+5} = 2$

9) $-\log_6(4x+7) + \log_6 x = 1$

10) $2 \cdot 5^{x-1} = 1$

Key

Math 70 Mixed Practice

Solve for exact answers, then find approximations with four decimal places.

$\log_5 4 = 3x-5$ $\quad \frac{\log_5(4)+5}{3} = x$

1) $\ln(3x) - \ln(x-3) = 2$

$\ln\left(\frac{3x}{x-3}\right) = 2$

$e^2 = \frac{3x}{x-3}$

$e^2x - 3e^2 = 3x$

$-3e^2 = 3x - e^2x$

$-3e^2 = x(3-e^2)$

$\boxed{x = \frac{-3e^2}{(3-e^2)}}$

$x \approx 5.05055$

$\boxed{5.0506}$

2) $3^{2x} = 7$

$\log 3^{2x} = \log 7$

$2x \log 3 = \log 7$

$x = \boxed{\frac{\log 7}{2\log 3}} = \boxed{\frac{\log 7}{\log 9}} \approx 0.88562$

$\approx \boxed{.8856}$

3) $\log_5 2 + \log_5 x = 2$

$\log_5 2x = 2$

$5^2 = 2x$

$25 = 2x$

$\frac{25}{2} = x$

$x = \boxed{12.5}$

4) $3^{2x+1} = 6$

$\log 3^{2x+1} = \log 6$

$(2x+1)\cdot \log 3 = \log 6$

$2x+1 = \frac{\log 6}{\log 3}$

$2x = \frac{\log 6}{\log 3} - 1$

$\boxed{x = \frac{\log 6}{2\log 3} - \frac{1}{2}}$

$x = .31546$

$x \approx \boxed{.3155}$

5) $\log(5x) - \log(x+1) = 4$

$\log \frac{5x}{x+1} = 4$

$10^4 = \frac{5x}{x+1}$

$10000(x+1) = 5x$

$10000x + 10000 = 5x$

$9995x = -10,000$

$x = \frac{-10,000}{9995} = \frac{-2000}{1999}$

NO SOLUTION

6) $5^{3x-5} = 4$

$\log 5^{3x-5} = \log 4$

$(3x-5)\cdot \log 5 = \log 4$

$3x-5 = \frac{\log 4}{\log 5}$

$3x = \frac{\log 4}{\log 5} + 5$

$x = \frac{\log 4}{3\log 5} + \frac{5}{3}$

$\boxed{x = \frac{\log 4}{\log 125} + \frac{5}{3}}$

$x \approx 1.95378$

$\boxed{x \approx 1.9538}$

7) $\log_2 x + \log_2 2x - 3 = 1$

$\log_2(x \cdot 2x) = 4$

$\log_2 2x^2 = 4$

$2^4 = 2x^2$

$16 = 2x^2$

$8 = x^2$

$x = +\sqrt{8}$ or ~~$-\sqrt{8}$~~

$x = \sqrt{8} =$

$\boxed{x = 2\sqrt{2}}$

$x = 2.82842$

$\boxed{x \approx 2.8284}$

8) $\frac{3\cdot 4^{x+5}}{3} = \frac{2}{3}$

$4^{x+5} = \frac{2}{3}$

$(x+5)\log 4 = \log 2/3$

$x+5 = \frac{\log 2/3}{\log 4}$

$\boxed{x = \frac{\log 2/3}{\log 4} - 5}$

$x = -5.29248$

$\boxed{x \approx -5.2925}$

9) $-\log_6(4x+7) + \log_6 x = 1$

$\log_6 x - \log_6(4x+7) = 1$

$\log_6\left(\frac{x}{4x+7}\right) = 1$

$6 = \frac{x}{4x+7}$

$6(4x+7) = x$

$24x + 42 = x$

$23x = -42$

$x = \frac{-42}{23}$

NO SOLUTION

10) $\frac{2\cdot 5^{x-1}}{2} = \frac{1}{2}$

$5^{x-1} = \frac{1}{2}$

$(x-1)\log 5 = \log(\frac{1}{2})$

$x-1 = \frac{\log(1/2)}{\log 5}$

$\boxed{x = \frac{\log .5}{\log 5} + 1}$

$x = .56932$

$\boxed{x \approx .5693}$

✱ Absolutely do not ever try to "divide by" a log.

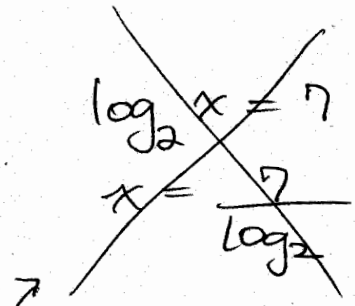
$\log_2 x = 7$

means

$2^7 = x$

$128 = x$

NOT



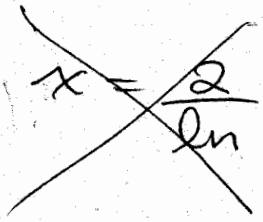
This says you have learned very little, so it will get zero credit.

Ditto $\ln x = 2$

means

$e^2 = x$

NOT



✱ ln or log without an argument is nonsense. We must always take the log or ln <u>of</u> something. ln(x) log(2)

<u>Our main strategies for equations:</u>

If we have a log, write an exponential.

If we have an exponential, take logs of both sides of the eqn.

9.7.47

Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to solve the compound interest problem.

Find how long it takes for $1500 to double if it is invested at 4% interest compounded monthly.

The money will double in value in approximately 8.7 years.
(Do not round until the final answer. Then round to the nearest tenth as needed.)

$P = 1500$

$A =$ double $1500 = 3000$

$r = .04$

$n = 12$ (monthly $\Rightarrow$ 12 times per year)

$t =$ unknown.

$$3000 = \underbrace{1500}_{\text{div by 1500 to isolate the exponential}}\underbrace{\left(1 + \frac{.04}{12}\right)^{12t}}_{\text{exponential}}$$

$$\frac{3000}{1500} = \left(1 + \frac{.04}{12}\right)^{12t}$$

Simplify

$$2 = \left(1 + \frac{.04}{12}\right)^{12t}$$

← Notice: Double reduces to 2. You can start at this step if you remember this.

Take logs

$$\log 2 = \log\left(1 + \frac{.04}{12}\right)^{12t}$$

Log property #3:

$$\log 2 = 12t \cdot \log\left(1 + \frac{.04}{12}\right)$$

← $1 + \frac{.04}{12} = \frac{301}{300}$ if you prefer

Isolate t: by dividing

$$\frac{\log 2}{12 \log\left(\frac{301}{300}\right)} = t$$

Calculate log(2)/(12* log(301/300)) enter

$t \approx 17.35 \Rightarrow \boxed{t \approx 17.4 \text{ yrs}}$

China is experiencing an annual growth rate of 0.606% In 2007, the population of China was 1,321,851,888. How long will it take for the population to be 1,500,000,000? Round to the nearest tenth of a year.

$y = y_0 e^{rt}$ Population model

y = population at time t = 1,500,000,000

y_0 = population at time 0 = 1,321,851,888

r = growth rate = 0.606% = 0.00606

$$1\,500\,000\,000 = 1,321,851,888\, e^{0.00606t}$$

sub/sub given values.

$$\frac{1\,500\,000\,000}{1\,321\,851\,888} = e^{0.00606t}$$

isolate exponential

$$\ln\left(\frac{1\,500\,000\,000}{1\,321\,851\,888}\right) = \ln e^{0.00606t}$$

$\ln x = \log_e x$ stuff

so RHS is $\log_e e$ = stuff

$$\ln\left(\frac{1\,500\,000\,000}{1\,321\,851\,888}\right) = 0.00606t$$

isolate t

$$\frac{\ln\left(\frac{1\,500\,000\,000}{1\,321\,851\,888}\right)}{0.00606} = t$$

$t \approx 20.86\ldots$ $t = 0$ means 2007

20.9 yrs $t = 20 \Rightarrow$ means 2027

during 2027 the population will pass 1.5 billion

9.7.49

Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to solve the compound interest problem.

Find how long it takes a $1400 investment to earn $500 interest if it is invested at 2% interest compounded quarterly.

$500 interest will be earned in approximately ☐ years. (Round to the nearest tenth.)

$P = 1400$

$A = 1400 + 500 = 1900$ ← "earn $500 interest" means add 500 to account

$r = .02$

$n = 4$ (quarterly ⇒ 4 times per year)

$t =$ unknown

$$1900 = 1400\left(1 + \frac{.02}{4}\right)^{4t}$$

Proceed as in previous problem
- isolate exponential
- take logs
- use log property #3
- isolate variable
- calculate & round.

$$\frac{1900}{1400} = (1.005)^{4t} \quad \text{or} \quad \left(\frac{201}{200}\right)^{4t}$$ - isolate exponential, - simplify the base of exponential

$$\frac{19}{14} = (1.005)^{4t} \quad \text{or} \quad \frac{19}{14} = \left(\frac{201}{200}\right)^{4t}$$

$$\log\left(\frac{19}{14}\right) = \log(1.005)^{4t} \quad \text{or} \quad \log\left(\frac{19}{14}\right) = \log\left(\frac{201}{200}\right)^{4t}$$ - take logs both sides

$$\log\left(\frac{19}{14}\right) = 4 \cdot t \cdot \log(1.005) \quad \text{or} \quad \log\left(\frac{19}{14}\right) = 4 \cdot t \cdot \log\left(\frac{201}{200}\right)$$ - log property

$$\frac{\log\left(\frac{19}{14}\right)}{[4\log(1.005)]} = t \quad \text{or} \quad \frac{\log\left(\frac{19}{14}\right)}{\left[4\log\left(\frac{201}{200}\right)\right]} = t$$ - isolate t

$15.307\ldots \approx t$

15.3 years $\approx t$

9.7.51 **Find how long it takes \$2000 to double if it is invested at 5% interest compounded semiannually.**

Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ to solve the compound interest problem.

It will take approximately ☐ years.

(Do not round until the final answer. Then round to the nearest tenth as needed.)

$P = 2000$

$A = 2(2000) = 4000$

$r = .05$

$n = 2$ (semi-annually means 2 times per year)

$t =$ unknown

$$4000 = 2000\left(1 + \frac{.05}{2}\right)^{2t}$$

proceed as in previous problem
- isolate exponential
- take logs
- use log property # 3
- isolate variable
- calculate & round.

$$\frac{4000}{2000} = \left(\frac{41}{40}\right)^{2t} \quad \text{or} \quad \frac{4000}{2000} = (1.025)^{2t}$$

- isolate exponential
- simplify the base of exponential.

$$2 = \left(\frac{41}{40}\right)^{2t} \quad \text{or} \quad 2 = (1.025)^{2t}$$

$$\log(2) = \log\left(\frac{41}{40}\right)^{2t} \quad \text{or} \quad \log(2) = \log(1.025)^{2t}$$

$$\log(2) = 2t \cdot \log\left(\frac{41}{40}\right) \quad \text{or} \quad \log(2) = 2 \cdot t \cdot \log(1.025)$$

$$\frac{\log(2)}{\left[2\log\left(\frac{41}{40}\right)\right]} = t \qquad \frac{\log(2)}{2\log(1.025)} = t$$

$14.0355\ldots \approx t$

$14.0 \approx t$ years